THE FAILURE OF THICK-WALLED CYLINDERS UNDER INTERNAL PRESSURE

By H. G. David*

[*Manuscript received August* 17, 1956]

Summary

A number of thick-walled hollow cylinders of Nickel-Chrome-Molybdenum steel have been subjected to internal pressures up to 12,000 atm at room temperature and their yield points determined. The results conform to the Leinss (1955) empirical relation between yield pressure and tensile yield stress.

I. Introduction

The design of high-pressure equipment almost invariably requires a knowledge of the maximum internal pressure a cylindrical vessel of given material and known wall thickness will support. The well-known theory of Lamé (1852) derives the stresses and strains in the walls of a thick cylinder subjected to internal pressure, assuming the material to be elastic. From this theory and various criteria for failure of the material, several different relations can be derived between the internal pressure causing failure and the ratio K of outer to inner diameters of the cylinder. Even the most optimistic of these leads to the conclusion that an infinitely thick cylinder cannot support an internal pressure greater than the ultimate tensile stress for the material.

It has long been known, however, from the work of Bridgman (1914*a*) and others, that steel cylinders for which K is about 9 will support pressures up to 20,000 atm, well above the ultimate tensile stress for the steel. The layers of metal near the bore of such cylinders are highly stressed and become plastic but the cylinder does not fail until the plastic-elastic boundary has moved some distance from the bore.

The existence of overstrained metal near the bore is utilized in the autofrettage process for giving greater strength to gun-barrels; the theory and practice of this process have been described by Macrae (1930) and others. Manning (1945) points out that this analysis is restricted to the situation in which the degree of overstrain is small, and has developed a theory based on stress-strain relations obtained from torsion tests in which the degree of overstrain is large.

Recently, Crossland and Bones (1955*a*, 1955*b*) have confirmed Manning's theory by carrying out bursting tests to 6000 atm on mild steel cylinders. Many consider, however, that computation of bursting pressures from Manning's formulae is laborious and complex and there is a strong desire to relate bursting pressure to the results of tensile rather than torsion tests. Leinss (1955) pointed out that the results of Crossland and Bones can be represented by the relation

$$\frac{(K-1)\sigma_u}{P_b} = \alpha = \alpha_1 + \beta(K-1), \quad \ldots\ldots\ldots\ldots\ldots (1)$$

---

* Division of Industrial Chemistry, C.S.I.R.O., High Pressure Laboratory, University of Sydney.

where K is the ratio of outer to inner diameters of a cylinder,

P_b is the bursting pressure of the cylinder,

σ_u is the ultimate tensile stress of the material,

α_1 and β are parameters independent of K.

Leinss showed that, since $(K-1)\sigma_u$ is the bursting pressure of a thin cylinder α_1 should be unity.

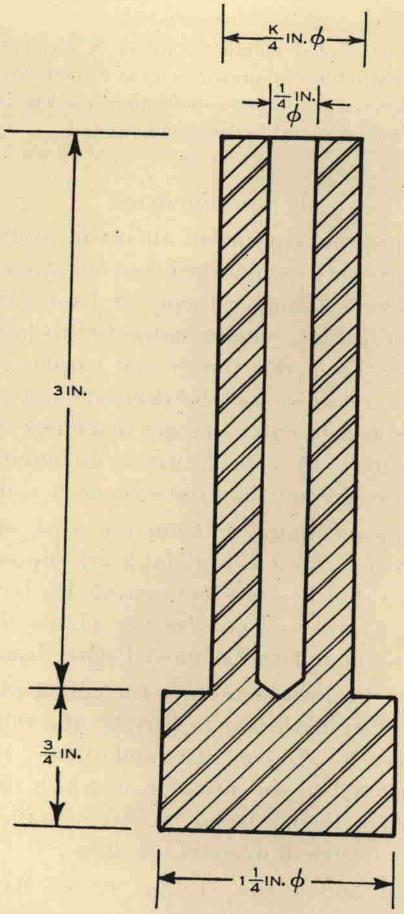

Fig. 1.—Test cylinder, longitudinal section.

The Leinss equation has the virtue of simplicity and provides the desired relationship between bursting pressure and tensile strength. Analysis of some published data shows that in all cases results can be represented by equations of the form (**1**) and it was decided to seek further experimental verification. At the same time it appeared desirable to investigate the variation of β with the tensile strength of the material.